TRINITY COLLEGE

Year 12 Methods Units 3/4
Test 4 2018

Section 1 Calculator Free

Logarithmic Functions

STUDENT'S NAME MARKING KEY

DATE: Thursday 19th July **TIME:** 30 minutes **MARKS:** 29

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser.

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (8 marks)

Differentiate the following with respect to x

(a) $\ln(2+4x+x^3)$ [2]

$$f'(x) = \frac{4+3x^2}{2+4x+x^3}$$

(b) $\ln\left(\frac{1}{(e^x+2x)^3}\right)$ [3]

$$= \ln\left[(e^x+2x)^{-3}\right]$$

$$\frac{d}{dx} = \frac{-3(e^x+2x)^{-4}(e^x+2)}{(e^x+2x)^{-3}}$$

$$= \frac{-3(e^x+2x)^3(e^x+2)}{(e^x+2x)^4}$$

or $\ln 1 - \ln(e^x+2x)^3$

$$= \ln 1 - 3\ln(e^x+2x)$$

$$\frac{d}{dx} = \frac{-3(e^x+2)}{e^x+2x}$$

(c) $\log_3(x^2-2x^3)$ [3]

$$= \ln\frac{x^2-2x^3}{\ln 3}$$

$$\frac{d}{dx} = \frac{2x-6x^2}{\ln 3(x^2-2x^3)}$$

2. (4 marks)

Determine the exact value of the gradient of the function $f(x) = \ln\dfrac{1+e^x}{1-e^x}$ when $x = \ln 2$.

$$f(x) = \ln(1+e^x) - \ln(1-e^x)$$

$$f'(x) = \frac{e^x}{1+e^x} + \frac{e^x}{1-e^x}$$

$$= -\frac{4}{3}$$

$$f'(\ln(2)) = \frac{e^{\ln 2}}{1+e^{\ln 2}} + \frac{e^{\ln 2}}{1-e^{\ln 2}}$$

$$= \frac{2}{3} + \frac{2}{-1}$$

3. (5 marks)

Determine

(a) $\displaystyle\int \frac{-4x^2}{2x^3-5}\,dx$ [2]

$$f(x) = 2x^3 - 5$$
$$f'(x) = 6x^2$$

$$\frac{-4}{6}\int \frac{6x^2}{2x^3-5}\,dx$$

$$= \frac{-2\ln|2x^3-5|}{3} + c$$

(b) $\displaystyle\int \tan(1-2x)\,dx$ [3]

$$= \int \frac{\sin(1-2x)}{\cos(1-2x)}\,dx$$

$$f(x) = \cos(1-2x)$$
$$f'(x) = 2\sin(1-2x)$$

$$= \frac{1}{2}\int \frac{2\sin(1-2x)}{\cos(1-2x)}\,dx$$

$$= \frac{\ln|\cos(1-2x)|}{2} + c$$

4. (5 marks)

Determine the exact area enclosed between $f(x) = x^2$ and $g(x) = 6 - \frac{6}{x+1}$ in the first quadrant.

$$x^2 = 6 - \frac{6}{x+1}$$

$$x^2(x+1) = 6(x+1) - 6$$

$$x^3 + x^2 = 6x$$

$$x^3 + x^2 - 6x = 0$$

$$x(x^2 + x - 6) = 0$$

$$x(x+3)(x-2) = 0$$

$$x = 0, x = -3, x = 2.$$

$$A = \int_0^2 6 - \frac{6}{x+1}\, dx - \int_0^2 x^2\, dx$$

$$= \int_0^2 6 - \frac{6}{x+1} - x^2\, dx$$

$$= \left[6x - 6\ln|x+1| - \frac{x^3}{3} \right]_0^2$$

$$= -6\ln 3 + \frac{28}{3} \text{ units}^2$$

5. (4 marks)

Given $\log_2 3 = a$ and $\log_2 5 = b$, determine in terms of a and b

(a) $\log_2 75$ [2]

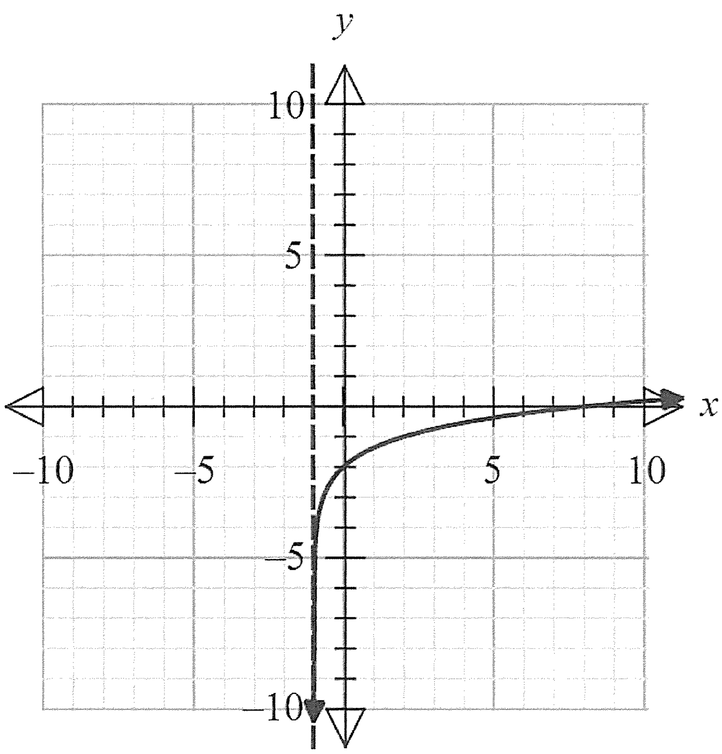
$a + 2b$

(b) $\log_2 2.5$ [2]

$b - 1$

6. (2 marks)

Determine the equation of the function shown below.



$f(x) = \log_3(x+1) - 2$

TRINITY COLLEGE

Year 12 Methods Units 3/4
Test 4 2018

Section 2 Calculator Assumed
Logarithmic Functions

STUDENT'S NAME MARKING KEY

DATE: Thursday 19th July **TIME:** 15 minutes **MARKS**: 14

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser.
Special Items: Up to three (3) approved calculators. One side A4 page of notes.

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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6. (3 marks)

A scale used to measure the intensity of earthquakes is known as the Richter Scale. The Richter scale is defined by the formula $R = \log\left(\frac{A}{A_0}\right)$ where A is the measure of the amplitude/intensity of the earthquake wave and A_0 is the amplitude/intensity of a standard wave.

A recent earthquake measured 6.8 on the Richter scale. How many times more intense was this earthquake than an earthquake that measured 4.3 on the Richter scale?

$6.8 = \log\left(\frac{A_1}{A_0}\right) \qquad 4.3 = \log\left(\frac{A_2}{A_0}\right)$

$\therefore \quad 2.5 = \log\left(\frac{A_1}{A_0}\right) - \log\left(\frac{A_2}{A_0}\right)$

$2.5 = \log(A_1) - \log(A_0) - [\log A_2 - \log A_0]$

$2.5 = \log \frac{A_1}{A_2}$

$\frac{A_1}{A_2} = 10^{2.5}$

$\frac{A_1}{A_2} = 316.23 \qquad \therefore A_1 \approx 316 A_2$

7. (7 marks)

Luigi's farm currently produces 10.1 tonnes of barley annually. Over an extended period of drought, he has found that the productivity of his land is decreasing but at a slowing rate. He decides that he will keep his barley farm until annual productivity reaches 0, so he uses a logarithmic function to model the annual productivity of his land t years from now.

(a) Using the model $P(t) = A + k\ln(t+1)$ where $P(t)$ is the annual productivity in tonnes after t years, solve for A and k if production drops to 7 tonnes after 1 year. [3]

$t = 0 \quad : \quad 10.1 = A + K\ln(1)$

$t = 1 \quad : \quad 7 = A + K\ln(2)$

$A = 10.1 \qquad K = \dfrac{-31}{10\ln(2)}$

(b) Assuming Luigi sells at the end of the year that the farms productivity reaches zero, determine after how many years he will sell his farm. [2]

$P(t) = 10.1 + \dfrac{-31}{10\ln(2)} \ln(t+1)$

$P(t) = 0$

$t = 8.56 \quad \sim 9$ years.

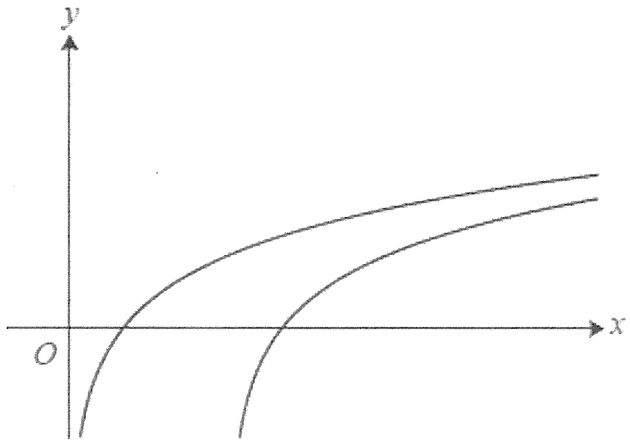
(c) At the end of the last year that Luigi runs the farm, at what rate will annual productivity be decreasing? [2]

$P(t) = \dfrac{-31}{10(x+1)\ln(2)}$

$P'(9) = -0.447$

8. (4 marks)

The diagram below shows the curves $y = \log_2 x$ and $y = \log_2(x-3)$.



(a) Describe the geometrical transformation that transforms the curve $y = \log_2 x$ to the curve $y = \log_2(x-3)$. [1]

$f(x-3)$

translation 3 units right parallel to x-axis

(b) The point P lines on $y = \log_2 x$ and has an x-coordinate of c. The point Q lies on $y = \log_2(x-3)$ and also has an x-coordinate of c. Given that the distance PQ is 4 units determine the exact value of c. [3]

$$\log_2 c - \log_2 (c-3) = 4$$

$$\therefore \log_2 \frac{c}{c-3} = 4$$

$$c = \frac{16}{5}$$